P.G. Semester-II Examination, 2023
MATHEMATICS

Course ID : 22155 **Course Code : MATH205C(IA)**

Course Title : Integral Transforms & Computational Methods for PDEs

Time : 2 Hours Full Marks : 32

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meaning.

GROUP–A

1. Answer any **two** of the following questions: 8×2=16

a) i) Prove that, for any a which is real,

$$\Im\left[f(at);t\to\xi\right]=\frac{1}{|a|}\Im\left[f(t);t\to\frac{\xi}{a}\right].$$

ii) Prove that,

$$\Im\left[e^{-t^2};t\to\xi\right]=\frac{1}{\sqrt{2}}e^{-\frac{1}{4}\xi^2}.$$ 4+4

b) Using the Fourier transform solve the following two dimensional Laplace equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ in $-\infty\le x<\infty$ and $y>0$ with $u(x,0)=f(x)\,\forall\,x\in(-\infty,\infty)$ and $u(x,y)\to 0$ as $x^2+y^2\to\infty$. 8

c) i) State the convolution theorem for Laplace transform.

ii) By using Laplace transform, solve the IVP

$\frac{d^2y}{dx^2}+6\frac{dy}{dx}+9y=\sin x$; with $y(0)=1$ and $y'(0)=0$. 2+6

GROUP–B

2. Answer any **two** of the following questions: 8×2=16

a) i) Use the general technique to construct finite difference discretization of

$$\left[\frac{\partial u}{\partial x}\right]_i$$

using a 3-point central difference formula. What is the truncation error (T.E.) of this formula? 3+1

ii) Construct the FTCS-scheme (forward in time and central in space scheme) for one dimensional transport equation

$$\frac{\partial u}{\partial t}+a\frac{\partial u}{\partial x}=\alpha\frac{\partial^2 u}{\partial x^2}$$

where a and α are constants. 4

b) Consider the 2D Laplace equation in a square

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$$

in $-1\le x\le 1;\ -1\le y\le 1$, with

$$u(-1,\ y)=u(1,\ y)=u(x,\ -1)=u(x,\ 1)=1.$$

Derive the system of algebraic equations resulting from the finite difference discretization using a 3-point central difference formula with grid size $\Delta x=\Delta y=0.5$. 8

c) Consider the one dimensional linear convection equation

$$\frac{\partial u}{\partial t}+a\frac{\partial u}{\partial x}=0$$

(a being constant) with suitable initial condition.

i) Show that the FTCS-scheme for this equation is: $u_i^{n+1}=u_i^n-\frac{1}{2}c\left(u_{i+1}^n-u_{i-1}^n\right)$, with $c=a\frac{\Delta t}{\Delta x}$. 3

ii) Show that the scheme is consistent with the above mentioned convection equation with a truncation error (T.E.) of the order of $O\left(\Delta t,\ \Delta x^2\right)$. 5

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